Input matching with a physical resistor

Input matching could be done with a physical 50Ω resistor. This would be sufficient to avoid most reflections e.g. when connecting a commercial signal generator to a DUT that is of higher input impedance. The typical input capacitance could also be neutralized with a shunt inductor so as to resonate at the wanted frequency. How to compute the noise factor of this circuit? Recall the definition of F = $vn_{O,TOT}^2/vn_{O,RS}^2$ where the first term in the total output noise voltage power spectral density (PSD) and the second the output noise voltage PSD due to Rs, the source resistance. The latter is simply 4kTRs multiplied by the square of the voltage divider gain RL/(Rs+RL).

$$V_{n,O,Rs}^2 = 4kT \cdot R_S \cdot \left(\frac{R_L}{R_S + R_L}\right)^2$$

To compute the total output noise, we apply the superposition principle after shorting all voltage sources and leaving open all current ones except for the one we are considering (we may directly sum the power contributions of the PSDs as long as there is no correlations between the noise sources which is the case if they are independent; or in other terms, if the latter condition is satisfied, their cross product is 0). RS and RL are thus in parallel each with a 4kTR equivalent noise voltage PSD. We may compute the equivalent output noise voltage PSD in two ways:

1. Place the voltage noise PSD of each resistor near their ground terminal rather than at the Vin node; apply in turn the correct squared voltage gain to compute the equivalent voltage noise density at node Vin; sum the two contributions. The contribution of Rs is exactly as calculated above, but the contribution of RL sees a different voltage divider radio Rs/(Rs+RL)

$$V_{n,O,TOT}^2 = 4kT \cdot R_S \cdot \left(\frac{R_L}{R_S + R_L}\right)^2 + 4kT \cdot R_L \cdot \left(\frac{R_S}{R_S + R_L}\right)^2 = 4kT \cdot \frac{R_S {R_L}^2 + R_L {R_S}^2}{(R_S + R_L)^2}$$
$$= 4kT \cdot \left(\frac{R_S R_L}{R_S + R_L}\right) = 4kT \cdot \left(R_S + R_L \cdot \frac{R_S^2}{R_L^2}\right) \cdot \left(\frac{R_L}{R_S + R_L}\right)^2$$

   The last form of the output noise eliminates the different divider ratio by multiplying the voltage noise PSD of RL by the square of Rs/RL, the equivalent square of the voltage gain needed to transfer the RL noise to Rs and use then the same voltage divider factor

2. Express the resistor noise as a current PSD given by 4kT/R (indeed the power PSD is also 4kT and thus equivalent for both representations); sum the current PSD; convert back to a voltage noise PSD after multiplying by the square of the equivalent resistance

$$V_{n,O,TOT}^2 = 4kT \cdot \left(\frac{1}{R_S} + \frac{1}{R_L}\right) \cdot \left(\frac{R_S R_L}{R_S + R_L}\right)^2 = 4kT \cdot \left(\frac{R_S R_L}{R_S + R_L}\right)$$

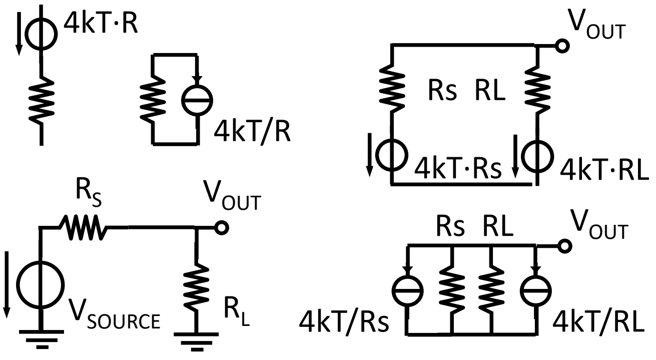

Figure 1. Bottom left : input matching with RL; top left: two equivalent representation for resistor noise as vn² [V²/Hz] or in² [A²/Hz]; top right: equivalent noise circuit according to 1); bottom right: ibid according to 2)

The noise factor is simply given by the ratio of the two computed output voltage PSD

$$F = \frac{V_{n,O,TOT}^2}{V_{n,O,Rs}^2} = \frac{4kT \cdot \left(\frac{1}{R_S} + \frac{1}{R_L}\right) \cdot \left(\frac{R_S \cdot R_L}{R_S + R_L}\right)^2}{4kT \cdot R_S \cdot \left(\frac{R_L}{R_S + R_L}\right)^2} = \frac{4kT \cdot \left(\frac{1}{R_S} + \frac{1}{R_L}\right) \cdot R_S^2 \cdot \left(\frac{R_L}{R_S + R_L}\right)^2}{4kT \cdot R_S \cdot \left(\frac{R_L}{R_S + R_L}\right)^2}$$

$$= \frac{4kT \cdot R_S + \frac{4kT}{R_L} \cdot R_S^2}{4kT \cdot R_S} = 1 + \frac{R_S}{R_L}$$

F is thus 2 with input match condition (NF=3dB). The noise is doubled since we have 2 equal contributors. Alternatively, F could be made close to 1 (no added noise) is RL goes to ∞. However this corresponds to an open circuit with only Rs thus no additional noise source, hence F=1. Conversely, presenting an input impedance lower than Rs results in a higher noise factor, which goes to ∞ as RL goes towards a short.

The condition for maximum power transfer and minimum noise factor is thus not achievable simultaneously. Usually, the power transfer is maximized while the designers try to present a matched input impedance without the associated noise. We will study this shortly.